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Effects of phase decoherence on the entanglement of a two-qubit anisotropic Heisenberg XYZ chain with an in-plane magnetic field

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Abstract. We investigate the phase decoherence effects on the entanglement of a two-qubit anisotropic Heisenberg XYZ model with a nonuniform magnetic field in the x–z-plane. As a measure of the entanglement, the concurrence of the system is calculated. It is shown that when the magnetic field is along the z-axis, the nonuniform and uniform components of the field have no influence on the entanglement for the cases of $\rho_1 = |00\rangle\langle 00|$ and $\rho_2 = |01\rangle\langle 01|$, respectively. But when the magnetic field is not along the z-axis, both the uniform and the nonuniform components of the field will introduce the decoherence effects. It is found that the effects of the Heisenberg chain's anisotropy in the Z-direction on the entanglement are dependent on the direction of the field. Moreover, the larger the initial concurrence is, the higher value it will exhibit during the time evolution of the system for a proper set of the parameters ν, Δ, θ, γ, B and b.

PACS. 03.67.-a Quantum information – 03.65.Ud Entanglement and quantum nonlocality (e.g. EPR paradox, Bell's inequalities, GHZ states, etc.) – 75.10.Jm Quantized spin models

QICS. 01.30.+r Quantum states and dynamics as a resource for information processing – 03.70.+c Entanglement versus correlation – 13.10.+n Effects of noise and imperfections

1 Introduction

Entanglement is one of the most novel features of quantum mechanics and it can constitute a fundamental resource for quantum computation and quantum information processing [1,2]. In recent years, the entanglement in various physical systems has been extensively explored. As a simple but realistic solid-state system, the Heisenberg model is an ideal candidate for the generation and the manipulation of entangled state. The model has been used to simulate quantum dots [3,4], nuclear spins [5], cavity QED [6,7] and optical lattices [8]. By suitable coding, the Heisenberg interaction alone can be used for quantum computation [9, 10]. Since the Heisenberg model is such a practical prototype, the entanglement in the one-dimensional Heisenberg chains has been studied in many works which can be separated into at least three categories, namely, those that study the infinite spin chains with at times particular attention to quantum phase transition [11], those that study the pairwise thermal entanglement in the n-qubit spin chains [12] and those that study the two-qubit Heisenberg spin chains [13,14].

It should be noted that the real quantum systems will be unavoidably connected with the surrounding environments, which can lead to decoherence. Decoherence is a quantum phenomenon that the quantum coherence is automatically destroyed when a quantum system evolves if the system has interaction with an environment. So it is of great importance to study the effects of environmental noise on the entanglement of the quantum system in view of the practice. By far, the quantum decoherence effects on the entanglement of the Heisenberg chains have been considered by a few authors. Li and Xu [15] discussed the Heisenberg XY chain with phase decoherence. They found that the neighbor pairwise entanglement of the XY chain with odd number length is more robust against phase decoherence than that with even number length. Li et al. [16] also studied the magnetic impurity effects on the entanglement of the three-qubit Heisenberg XY chain with intrinsic decoherence. It is shown that the neighbor pairwise entanglement decays faster than the next to neighbor pairwise entanglement in the case without field impurity and that the magnetic impurity in middle qubit can protect the neighbor pairwise entanglement from the intrinsic decoherence. Shao et al. [17] considered the influence of intrinsic decoherence on the entanglement of a

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two-qubit quantum Heisenberg XYZ chain. Subsequently, Wang et al. [18] investigated the entanglement evolution of the two-qubit system via the Heisenberg XY interaction in the presence of decoherence. They studied the decoherence influence on the entanglement due to population relaxation and thermal effects and then identified the contributions of global and local coherence to the steady state entanglement.

In the solid state devices for physical implementation of qubits [3,5,6,9,19], the interaction between the two qubits is governed by an isotropic Heisenberg Hamiltonian. But the anisotropy is just an approximation in the solid-matter system because the spin-orbit coupling always introduces the perturbations which will break the isotropy of the system. Therefore, in the theoretical analysis it is desirable to study the influence of the anisotropy in the Heisenberg chains. Furthermore, the inhomogeneous Zeeman coupling is always possible in any solid state construction of qubits [20]. As a consequence, the nonuniformity of the external magnetic field should be included in the Hamiltonian of the spin chain model. Recently, the influences of both the anisotropy of the Heisenberg model and the inhomogeneity of the external magnetic field on the thermal entanglement of the spin chains have been studied in reference [14], where the spin systems were not considered to be subject to decoherence effect. As far as we know, although the authors of references [15–18] have investigated the entanglement evolution of the Heisenberg spin model with decoherence effect, the anisotropy of spin chains and the general cases of magnetic fields were less studied yet. Based on the above analysis, we think it is worth examining the combined influences of the decoherence effect, the anisotropy of the Heisenberg chains and the general cases of magnetic fields on the entanglement of the Heisenberg chains. So in this paper we attempt to investigate the phase decoherence effects on the entanglement of a two-qubit anisotropic Heisenberg XYZ chain with a nonuniform magnetic field in the x–z-plane by employing the concurrence. Our results show that both the direction of the field and the initial state of the system play an important role during the time evolution of the concurrence in the presence of phase decoherence.

2 Model and concurrence

The Hamiltonian of the two-qubit anisotropic Heisenberg XYZ chain with an inhomogeneous magnetic field in the x–z-plane can be written as

$$H = \frac{1}{2}[(1+\nu)J\sigma_1^x \otimes \sigma_2^x + (1-\nu)J\sigma_1^y \otimes \sigma_2^y + \Delta J\sigma_1^z \otimes \sigma_2^z + (B+b)(\sigma_1^x \cos\theta + \sigma_1^z \sin\theta) + (B-b)(\sigma_2^x \cos\theta + \sigma_2^z \sin\theta)], \tag{1}$$

where σ_n^α ($\alpha = x, y, z$; $n = 1, 2$) are the Pauli matrices of the nth qubit. J is the real coupling constant for the spin interaction. The chain is said to be antiferromagnetic for $J > 0$ and ferromagnetic for $J < 0$. The parameter ν measures the anisotropy (partial anisotropy) in the XY-plane and Δ measures the anisotropy in the Z-direction. θ is the angle between the direction of the magnetic field and the x-axis. The uniform magnetic field $B \geq 0$ is restricted and b denotes the degree of inhomogeneity of the field at the two spin sites.

In the situation of a pure phase decoherence, the master equation describing the time-dependent dynamical evolution of the system under the Markovian approximation is given by [21]

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] - \frac{\gamma}{2}[H,[H,\rho(t)]], \tag{2}$$

where γ is the phase decoherence rate. Note that equation (2) has the similar form to the equation which has been used to depict the intrinsic decoherence [22]. The formal solution of the above master equation can be expressed as [23]

$$\rho(t) = \sum_{k=0}^{\infty} \frac{(\gamma t)^k}{k!} M^k \rho(0) M^{+k}, \tag{3}$$

where $\rho(0)$ is the initial density operator of the system and M^k is defined by

$$M^k = H^k \exp(-iHt) \exp\left(-\frac{\gamma t}{2}H^2\right). \tag{4}$$

If the two-qubit anisotropic Heisenberg XYZ chain described by equation (1) is taken into account, the time evolution of the density operator of the system is shown as

$$\rho(t) = \sum_{mn} \exp\left[-\frac{\gamma t}{2}(E_m - E_n)^2 - i(E_m - E_n)t\right] \times \langle\phi_m|\,\rho(0)\,|\phi_n\rangle\,|\phi_m\rangle\,\langle\phi_n|, \tag{5}$$

where $E_{m,n}$ and $|\phi_{m,n}\rangle$ are the eigenvalues and the corresponding eigenvectors of Hamiltonian, respectively.

In this paper, we adopt the concurrence C as a measure of the entanglement. The concurrence related to the density operator ρ of a pair of qubits is defined by [24]

$$C(\rho) = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \tag{6}$$

where the quantities $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the square roots of the eigenvalues of the "spin-flipped" density matrix operator

$$\varrho = \rho(\sigma_1^y \otimes \sigma_2^y)\rho^*(\sigma_1^y \otimes \sigma_2^y), \tag{7}$$

where the asterisk indicates the complex conjugation. The concurrence $C = 0$ corresponds to a separable state and $C = 1$ to a maximally entangled state. Nonzero concurrence means that the two qubits are entangled.

3 Decoherence effects on the entanglement of the XYZ model with the magnetic field along the z-axis

In this section, we will consider the phase decoherence effects on the entanglement of the two-qubit anisotropic

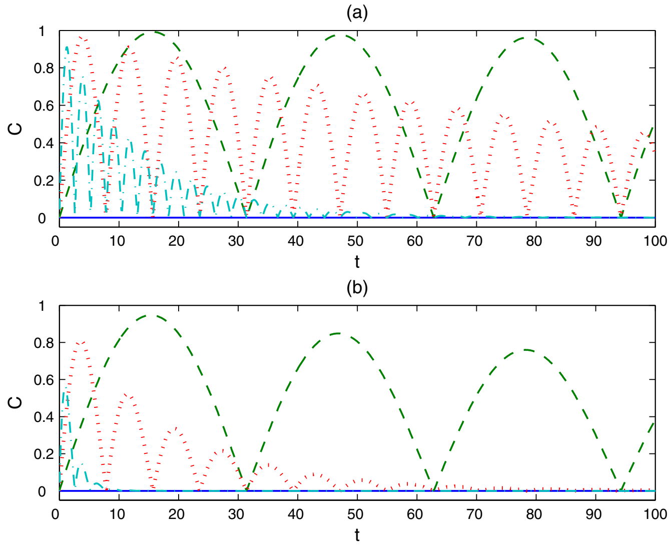

Fig. 1. (Color online) Concurrence of the two-qubit Heisenberg XYZ model without a magnetic field is plotted as a function of time t for different anisotropic parameters ν and different phase decoherence rates γ, where (a) $\gamma = 0.1$, (b) $\gamma = 0.7$. Here the solid line corresponds to $\nu = 0$, the dashed line to $\nu = 0.05$, the dotted line to $\nu = 0.2$, the dash-dotted line to $\nu = 0.6$. Here and after, the coupling constant J is set to one.

Heisenberg XYZ chain with a nonuniform magnetic field along the z-axis. When the field is along the z-axis, the Hamiltonian of the two-qubit Heisenberg XYZ chain is given by equation (1) with $\theta = \pi/2$. In the standard basis $\{|11\rangle, |10\rangle, |01\rangle, |00\rangle\}$, the eigenvectors of the Hamiltonian can be derived as

$$\begin{aligned}
|\Psi_1^z\rangle &= \frac{1}{\sqrt{1+\xi^2/J^2}}(\xi|10\rangle/J + |01\rangle),\\
|\Psi_2^z\rangle &= \frac{1}{\sqrt{1+\zeta^2/J^2}}(\zeta|10\rangle/J + |01\rangle),\\
|\Psi_3^z\rangle &= \frac{1}{\sqrt{1+\mu^2/(\nu J)^2}}(\mu|11\rangle/\nu J + |00\rangle),\\
|\Psi_4^z\rangle &= \frac{1}{\sqrt{1+\kappa^2/(\nu J)^2}}(\kappa|11\rangle/\nu J + |00\rangle),
\end{aligned} \tag{8}$$

with the corresponding eigenvalues:

$$\begin{aligned}
E_1^z &= -\frac{\Delta J}{2} - \delta, \quad E_2^z = -\frac{\Delta J}{2} + \delta,\\
E_3^z &= \frac{\Delta J}{2} - \sigma, \quad E_4^z = \frac{\Delta J}{2} + \sigma,
\end{aligned} \tag{9}$$

where $\delta = \sqrt{J^2 + b^2}$, $\sigma = \sqrt{(\nu J)^2 + B^2}$, $\xi = b - \delta$, $\zeta = b + \delta$, $\mu = B - \sigma$ and $\kappa = B + \sigma$.

First, we consider the case that the qubits 1 and 2 are both initially in the spin-down states, i.e., they are initially in the unentangled state $\rho_1(0) = |00\rangle\langle 00|$. Based on equations (5, 8, 9), the time evolution of the density operator of the system can be obtained as

$$\rho_1(t) = \omega_1 |11\rangle\langle 11| + \varepsilon_1 |11\rangle\langle 00| + \varepsilon_1^* |00\rangle\langle 11| + \varpi_1 |00\rangle\langle 00|, \tag{10}$$

where

$$\begin{aligned}
\omega_1 &= \left[\frac{\mu/\nu J}{1+\mu^2/(\nu J)^2}\right]^2\\
&+ \exp(-2\gamma\sigma^2 t + 2i\sigma t)\frac{\mu/\nu J}{1+\mu^2/(\nu J)^2}\frac{\kappa/\nu J}{1+\kappa^2/(\nu J)^2}\\
&+ \left[\frac{\kappa/\nu J}{1+\kappa^2/(\nu J)^2}\right]^2\\
&+ \exp(-2\gamma\sigma^2 t - 2i\sigma t)\frac{\mu/\nu J}{1+\mu^2/(\nu J)^2}\frac{\kappa/\nu J}{1+\kappa^2/(\nu J)^2},\\
\varepsilon_1 &= \frac{\mu/\nu J}{[1+\mu^2/(\nu J)^2]^2}\\
&+ \exp(-2\gamma\sigma^2 t - 2i\sigma t)\frac{1}{1+\mu^2/(\nu J)^2}\frac{\kappa/\nu J}{1+\kappa^2/(\nu J)^2}\\
&+ \frac{\kappa/\nu J}{[1+\kappa^2/(\nu J)^2]^2}\\
&+ \exp(-2\gamma\sigma^2 t + 2i\sigma t)\frac{\mu/\nu J}{1+\mu^2/(\nu J)^2}\frac{1}{1+\kappa^2/(\nu J)^2},\\
\varpi_1 &= \left[\frac{1}{1+\mu^2/(\nu J)^2}\right]^2\\
&+ \exp(-2\gamma\sigma^2 t + 2i\sigma t)\frac{1}{1+\mu^2/(\nu J)^2}\frac{1}{1+\kappa^2/(\nu J)^2}\\
&+ \left[\frac{1}{1+\kappa^2/(\nu J)^2}\right]^2\\
&+ \exp(-2\gamma\sigma^2 t - 2i\sigma t)\frac{1}{1+\mu^2/(\nu J)^2}\frac{1}{1+\kappa^2/(\nu J)^2}.
\end{aligned} \tag{11}$$

In this case, according to equations (6, 7, 10), the concurrence of the system can be given by

$$\begin{aligned}
C(\rho_1) &= \sqrt{\frac{m_1 + q_1 + \sqrt{(m_1 - q_1)^2 + 4n_1p_1}}{2}}\\
&\quad - \sqrt{\frac{m_1 + q_1 - \sqrt{(m_1 - q_1)^2 + 4n_1p_1}}{2}},
\end{aligned} \tag{12}$$

where $m_1 = \varepsilon_1\varepsilon_1^* + \omega_1\varpi_1^*$, $n_1 = \varepsilon_1\omega_1^* + \omega_1\varepsilon_1$, $p_1 = \varpi_1\varepsilon_1^* + \varepsilon_1^*\varpi_1^*$, $q_1 = \varpi_1\omega_1^* + \varepsilon_1\varepsilon_1^*$. Note that the coefficients ω_1, ε_1 and ϖ_1 do not contain the parameters b and Δ, that is to say, both the nonuniformity of the magnetic field and the anisotropy in the Z-direction of the Heisenberg chain have no influence on the entanglement even if $\gamma \neq 0$.

In the following we will give the numerical results of the concurrence. In Figure 1 we plot the concurrence $C(\rho_1)$ as a function of time t for different ν with different phase decoherence rate γ. By comparing Figure 1a with 1b, it is easy to observe that the destructive effect of the decoherence becomes more visible after the decoherence rate γ is raised. Because b and Δ have no decoherence effect on the entanglement of the system and we set $B = 0$, so the decoherence effect is due to the Heisenberg chain's partial anisotropy which is measured by the parameter ν. From Figure 1, one can find that there is no decoherence effect

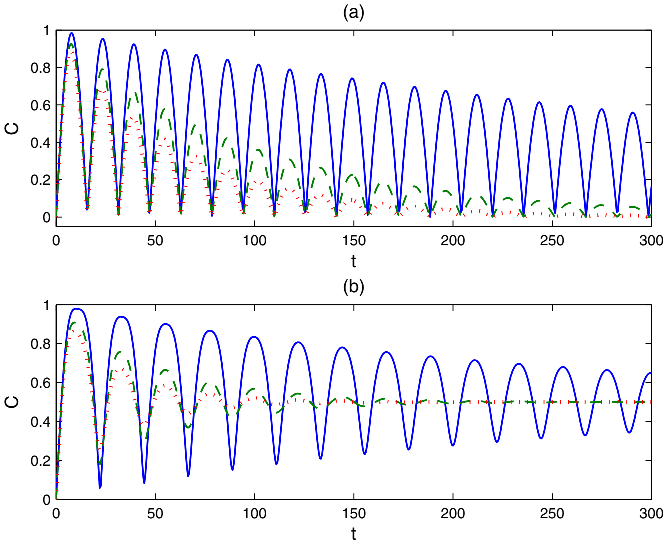

Fig. 2. (Color online) Concurrence of the two-qubit Heisenberg XYZ model with a uniform magnetic field is plotted as a function of time t for different phase decoherence rates γ, where (a) $B = 0$, (b) $B = 0.1$. Here the solid line corresponds to $\gamma = 0.1$, the dashed line to $\gamma = 0.5$, the dotted line to $\gamma = 0.8$. The anisotropic parameter $\nu = 0.1$.

if $\nu = 0$ (the x–y-plane is isotropic), but the decoherence effects will exist for the $\nu \neq 0$ case and the extreme value of the concurrence decays faster during the time evolution of the system if the partial parameter ν is raised. The decoherence case that the x–y-plane is isotropic can be understood as follows: when $\nu = 0$, the eigenvectors $|\Psi_3^z\rangle$ and $|\Psi_4^z\rangle$ respectively go to

$$|\Psi_3'^z\rangle = |00\rangle, \quad |\Psi_4'^z\rangle = |11\rangle, \tag{13}$$

with the corresponding eigenvalues:

$$E_3'^z = \frac{\Delta J}{2} - B, \quad E_4'^z = \frac{\Delta J}{2} + B. \tag{14}$$

In this situation, if the system is initially in the state $\rho_1'(0) = |a_0|^2 |11\rangle\langle 11| + |b_0|^2 |00\rangle\langle 00|$ ($|a_0|^2 + |b_0|^2 = 1$), the time evolution of the density operator of the system will remain as the initial state, i.e., $\rho_1'(t) = \rho_1'(0)$. Obviously, the state $\rho_1(0) = |00\rangle\langle 00|$ is a special case of $\rho_1'(0)$ with $|a_0|^2 = 0$ and $|b_0|^2 = 1$, that is the reason why the solid line in Figure 1 keep to zero with the evolution of time t.

In Figure 2 we depict the concurrence $C(\rho_1)$ changing with time t for different γ without the magnetic field B (for Fig. 2a) or with the magnetic field B (for Fig. 2b). One can see that the depletable effect of the phase decoherence on the concurrence increases with the increasing γ, which is consistent with the result of Figure 1. If there is no magnetic field, the concurrence is at last destroyed completely due to the decoherence effect, which can be seen from Figure 2a. In contrast to Figure 2a, after the magnetic field is given in Figure 2b, the concurrence for different γ will arrive at a steady-going nonzero value for the long-time case, which means that the uniform magnetic field B is a positive component to the entanglement when the partial anisotropic parameter of the system is at a fixed nonzero value. In references [16,17] they obtained the similar result that a proper external magnetic field can protect the entanglement from the destructive effect of the intrinsic decoherence.

Now we consider the situation that the qubits 1 and 2 are initially in another unentangled state $\rho_2(0) = |01\rangle\langle 01|$. In this case, the time evolution of the density operator of the system can be derived as

$$\rho_2(t) = \omega_2 |10\rangle\langle 10| + \varepsilon_2 |10\rangle\langle 01| + \varepsilon_2^* |01\rangle\langle 10| + \varpi_2 |01\rangle\langle 01|, \tag{15}$$

where

$$\begin{aligned}
\omega_2 = {} & \left[\frac{\xi/J}{1+\xi^2/J^2}\right]^2 \\
& + \exp(-2\gamma\delta^2 t + 2i\delta t)\frac{\xi/J}{1+\xi^2/J^2}\frac{\zeta/J}{1+\zeta^2/J^2} \\
& + \left[\frac{\zeta/J}{1+\zeta^2/J^2}\right]^2 \\
& + \exp(-2\gamma\delta^2 t - 2i\delta t)\frac{\xi/J}{1+\xi^2/J^2}\frac{\zeta/J}{1+\zeta^2/J^2}, \\
\varepsilon_2 = {} & \frac{\xi/J}{(1+\xi^2/J^2)^2} \\
& + \exp(-2\gamma\delta^2 t - 2i\delta t)\frac{1}{1+\xi^2/J^2}\frac{\zeta/J}{1+\zeta^2/J^2} \\
& + \frac{\zeta/J}{(1+\zeta^2/J^2)^2} \\
& + \exp(-2\gamma\delta^2 t + 2i\delta t)\frac{\xi/J}{1+\xi^2/J^2}\frac{1}{1+\zeta^2/J^2}, \\
\varpi_2 = {} & \left[\frac{1}{1+\xi^2/J^2}\right]^2 \\
& + \exp(-2\gamma\delta^2 t + 2i\delta t)\frac{1}{1+\xi^2/J^2}\frac{1}{1+\zeta^2/J^2} \\
& + \left[\frac{1}{1+\zeta^2/J^2}\right]^2 \\
& + \exp(-2\gamma\delta^2 t - 2i\delta t)\frac{1}{1+\xi^2/J^2}\frac{1}{1+\zeta^2/J^2}.
\end{aligned} \tag{16}$$

Then from equations (6, 7, 15), we can obtain the concurrence

$$\begin{aligned}
C(\rho_2) = {} & \sqrt{\frac{m_2 + q_2 + \sqrt{(m_2 - q_2)^2 + 4n_2p_2}}{2}} \\
& - \sqrt{\frac{m_2 + q_2 - \sqrt{(m_2 - q_2)^2 + 4n_2p_2}}{2}},
\end{aligned} \tag{17}$$

where $m_2 = \varepsilon_2\varepsilon_2^* + \omega_2\varpi_2^*$, $n_2 = \varepsilon_2\omega_2^* + \omega_2\varepsilon_2$, $p_2 = \varpi_2\varepsilon_2^* + \varepsilon_2^*\varpi_2^*$, $q_2 = \varpi_2\omega_2^* + \varepsilon_2\varepsilon_2^*$. What should be mentioned here is that, the coefficients ω_2, ε_2 and ϖ_2 are independent of the parameters B and Δ, which indicates

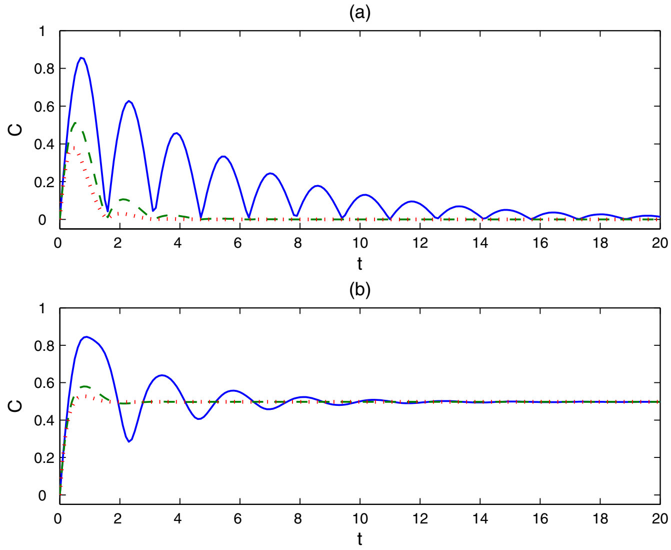

Fig. 3. (Color online) Concurrence of the two-qubit Heisenberg XYZ model with a nonuniform magnetic field is plotted as a function of time t for different phase decoherence rates γ, where (a) $b = 0$, (b) $b = 0.9$. Here the solid line corresponds to $\gamma = 0.1$, the dashed line to $\gamma = 0.5$, the dotted line to $\gamma = 0.8$. The anisotropic parameter $\nu = 0.1$.

that both the uniform component of the magnetic field and the anisotropy in the Z-direction of the Heisenberg chain have no effect on the entanglement.

In Figure 3 we display the concurrence $C(\rho_2)$ as a function of time t without (for Fig. 3a) or with (for Fig. 3b) the nonuniform magnetic field b. By comparing Figure 3 with Figure 2, we find that whether the magnetic field is given or not, the concurrence's oscillating time of the $\rho_2(0) = |01\rangle\langle 01|$ case is much shorter than that of the $\rho_1(0) = |00\rangle\langle 00|$ case. This indicates that the initial state of the system plays an important role in the time evolution of the entanglement of the system with phase decoherence. Besides, By comparing Figure 3b with Figure 2b, one can see that the effect of the nonuniform field b on the concurrence in the $\rho_2(0)$ case is similar to that of the uniform field B in the $\rho_1(0)$ case, i.e., the nonuniform field b can also weaken the destructive effect of the phase decoherence on the entanglement to some extent for the long-time case.

Since the magnetic field can protect the entanglement from being destroyed by the decoherence effect and generate a stationary entangled state of the two qubits for the long-time case, next we will focus on investigating how the field affect the concurrence $C(\rho_1)$ and $C(\rho_2)$ in the case of the long-time limit. As the time t approaches to infinite, the coefficients ω_1, ε_1 and ϖ_1 will respectively turn into

see equations (18) below.

According to equations (12) and (18), if the parameter ν is at its fixed value, the concurrence $C(\rho_1)$ is only influenced by the strength of the uniform magnetic field B $(J = 1)$. Similarly, in the case that the time t approximates infinite, the coefficients ω_2, ε_2 and ϖ_2 will respectively turn into

see equations (19) below.

From equations (17) and (19), it is easily to see that the concurrence $C(\rho_2)$ is only affected by the strength of the nonuniform magnetic field b $(J = 1)$ for the long-time case. In Figure 4 we plot the influences of both the uniform (Fig. 4a) and the nonuniform (Fig. 4b) magnetic field on the entanglement of the system in the presence of the phase decoherence for the long-time case. It is shown that with the increases of the strength of the magnetic field, the concurrence first increases from zero to a maximal value, then it begins to decay monotonously. From the figure, one can see that the maximal value of the concurrence $C_{\max}(\rho_1) \simeq 0.5$ appears at $B \simeq 0.1$, while $C_{\max}(\rho_2) \simeq 0.5$ appears at $b \simeq 1$, which means that the concurrence of the $\rho_1(0) = |00\rangle\langle 00|$ case is sensitive to the uniform magnetic field, but the concurrence of the $\rho_2(0) = |01\rangle\langle 01|$ case is not so sensible to the nonuniform magnetic field.

4 Decoherence effects on the entanglement of the XYZ model with the magnetic field along an arbitrary direction in the x–z-plane

After examining the phase decoherence effects on the entanglement of the two-qubit Heisenberg XYZ model with the magnetic field along the z-axis in the above section, now in this section, we will study the influence of the decoherence on the entanglement of the XYZ model with the field along an arbitrary direction in the x–z-plane.

$$\omega_1' = \frac{\nu^2 J^2}{2(B^2+\nu^2 J^2)}, \quad \varepsilon_1' = \frac{-2B}{\nu J[1+(2B^2+\nu^2J^2+2B\sqrt{B^2+\nu^2J^2})/\nu^2J^2][1+(2B^2+\nu^2J^2-2B\sqrt{B^2+\nu^2J^2})/\nu^2J^2]}$$
$$\varpi_1' = \frac{1}{[1+(2B^2+\nu^2J^2+2B\sqrt{B^2+\nu^2J^2})/\nu^2J^2]^2} + \frac{1}{[1+(2B^2+\nu^2J^2-2B\sqrt{B^2+\nu^2J^2})/\nu^2J^2]^2} \tag{18}$$

$$\omega_2' = \frac{J^2}{2(b^2+J^2)}, \quad \varepsilon_2' = \frac{-2b}{J[1+(2b^2+J^2+2b\sqrt{b^2+J^2})/J^2][1+(2b^2+J^2-2b\sqrt{b^2+J^2})/J^2]}$$
$$\varpi_2' = \frac{1}{[1+(2b^2+J^2+2b\sqrt{b^2+J^2})/J^2]^2} + \frac{1}{[1+(2b^2+J^2-2b\sqrt{b^2+J^2})/J^2]^2} \tag{19}$$

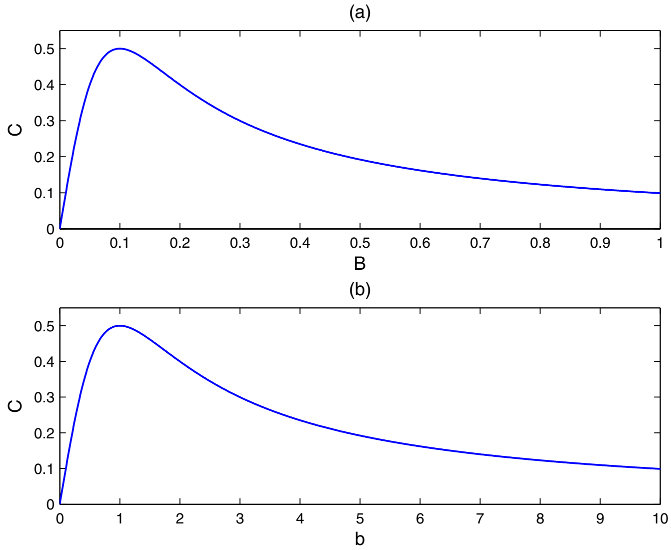

Fig. 4. Concurrence of the two-qubit Heisenberg XYZ model versus B and b for different initial states at the infinite time limit, where (a) $\rho_1(0) = |00\rangle\langle 00|$, (b) $\rho_2(0) = |01\rangle\langle 01|$. The anisotropic parameter $\nu = 0.1$.

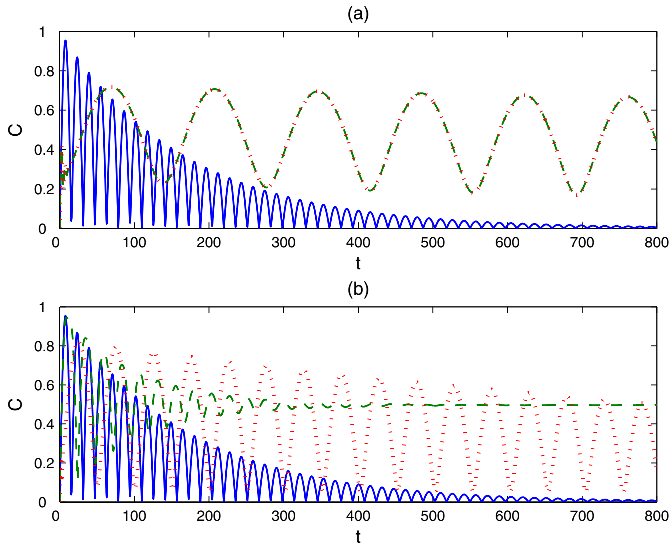

Fig. 5. (Color online) Concurrence of the two-qubit Heisenberg XYZ model with different magnetic fields is plotted as a function of time t, where (a) $\theta = 0$, the solid line corresponds to $B = 0$ and $b = 0$, the dashed line to $B = 0.5$ and $b = 0$, the dotted line to $B = 0$ and $b = 0.5$, (b) $\theta = \pi/3$, the solid line corresponds to $B = 0$ and $b = 0$, the dashed line to $B = 0.1$ and $b = 0$, the dotted line to $B = 0$ and $b = 1$. The other parameters are $\nu = 0.1$, $\Delta = 0$, $\gamma = 0.3$.

The Hamiltonian of the two-qubit anisotropic Heisenberg XYZ chain with an inhomogeneous magnetic field in the x–z-plane is given by equation (1). We do not list the analytical expressions of the eigenvectors and the eigenvalues of the Hamiltonian here because they are rather complicated.

In Figure 5 we plot the concurrence evolving with time t for different magnitudes and different directions of the magnetic field (the initial state of the system is supposed to be $\rho_1(0) = |00\rangle\langle 00|$). It can be seen that in the $\theta \neq \pi/2$ case, both the uniform component and the nonuniform component of the magnetic field will have effects on the entanglement during the decoherence process. In Figure 5a, the dashed line ($B = 0.5$, $b = 0$) and the dotted line ($B = 0$, $b = 0.5$) lap over with each other, which implies that for the case that the magnetic field is along the x-axis and $\Delta = 0$, whether the fields on the two qubits are in the same direction or in the opposite directions, the concurrence are in good agreement with each other if the strength of the fields on the two qubits are same. Moreover, by comparing the dotted line with the solid line in each graph of Figure 5, we find that a proper set of the magnitude and the direction of the magnetic field can greatly prolong the fluctuant time of the concurrence before the system reaches a steady state.

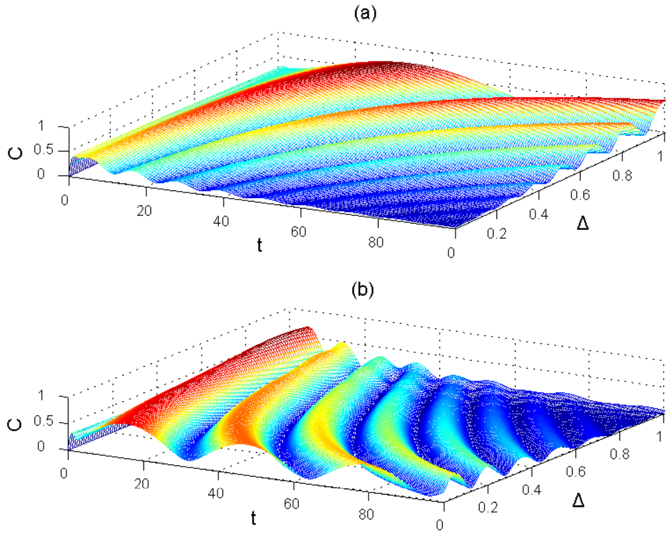

Fig. 6. (Color online) Concurrence of the two-qubit Heisenberg XYZ model with a nonuniform magnetic field is plotted versus t and Δ for different θ, where (a) $\theta = 0$, (b) $\theta = \pi/4$. The other parameters are $\nu = 0.2$, $\gamma = 0.6$, $B = 0$, $b = 1$.

In Figure 6 we consider the influence of the anisotropic parameter Δ on the entanglement in the presence of decoherence (the system is also assumed to be initially in $\rho_1(0) = |00\rangle\langle 00|$). From Figure 6a one can observe that the maximum concurrence the system can arrive at becomes larger after Δ is raised and that the concurrence at smaller Δ decays much rapidly than that at larger Δ. This means that the destructive effect of the phase decoherence on the entanglement is reduced by the introduction of the anisotropy in the Z-direction. But we notice that after the magnetic field is changed into another direction, the effect of parameter Δ on the entanglement with decoherence in Figure 6b is completely opposite to that in Figure 6a, which indicates that the influence of Δ on the decoherence process is dependent on the direction of the magnetic field.

In our previous discussions, we only considered that the two qubits 1 and 2 are supposed to be initially in the unentangled states $\rho_1(0) = |00\rangle\langle 00|$ and $\rho_2(0) = |01\rangle\langle 01|$.

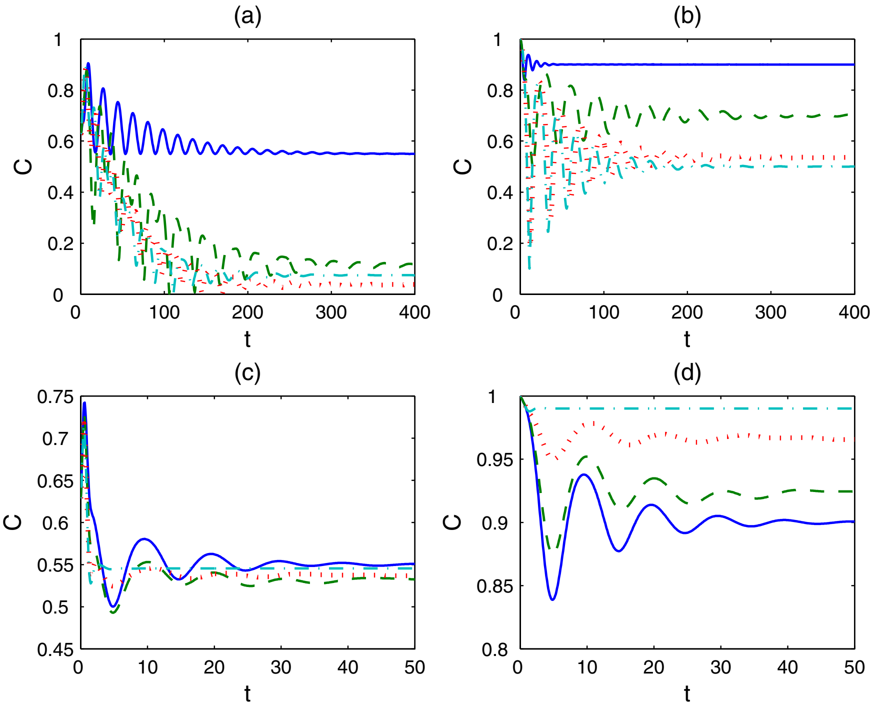

Fig. 7. (Color online) Concurrence of the two-qubit Heisenberg XYZ model with a nonuniform magnetic field is plotted versus t for different initial states, where (a) $a_1 = 1/3$, $b_1 = 2\sqrt{2}/3$, (b) $a_1 = \sqrt{2}/2$, $b_1 = \sqrt{2}/2$, (c) $a_2 = 1/3$, $b_2 = 2\sqrt{2}/3$, (d) $a_2 = \sqrt{2}/2$, $b_2 = \sqrt{2}/2$. Here the solid line corresponds to $\theta = 0$, the dashed line to $\theta = \pi/6$, the dotted line to $\theta = \pi/3$, the dash-dotted line to $\theta = \pi/2$. The other parameters are $\nu = 0.1$, $\Delta = 0.3$, $\gamma = 0.5$, $B = 0.1$, $b = 0.1$.

In what followed, the situation that the initial state of the system is an entangled state will be taken into account and we will concentrate on studying the influence of the initial concurrence of the two qubits on the time evolution of the entanglement of the system in the presence of the phase decoherence. In Figure 7 we plot the evolution of the concurrence whose initial state is an entangled state. Figures 7a and 7b depict the case that the qubits 1 and 2 are initially in the entangled state $|\Phi(0)\rangle\langle\Phi(0)|$ ($|\Phi(0)\rangle = a_1|00\rangle + b_1|11\rangle$, $|a_1|^2 + |b_1|^2 = 1$), while Figures 7c and 7d represent the case that qubits 1 and 2 are entangled in the state $|\Psi(0)\rangle\langle\Psi(0)|$ ($|\Psi(0)\rangle = a_2|01\rangle + b_2|01\rangle$, $|a_2|^2 + |b_2|^2 = 1$) at the beginning. Obviously, the concurrence of the system which is initially in the state $|\Psi(0)\rangle\langle\Psi(0)|$ decays more rapidly than that of which is initially in the state $|\Phi(0)\rangle\langle\Phi(0)|$ before they reach their steady values. By comparing Figure 7a with Figure 7b and comparing Figure 7c with Figure 7d, we find that under the condition of a fixed set of the parameters ν, Δ, θ, γ, B and b, the larger the initial concurrence of the two qubits, the higher value the concurrence will display in the time evolution of the system with phase decoherence. This denotes that the initial concurrence of the two qubits is of great significance on the entanglement during the whole evolution process of the system.

5 Conclusions

In summary, by calculating the concurrence, we have provided a detailed analytical and numerical analysis of phase decoherence for a two-qubit anisotropic Heisenberg XYZ model system with a nonuniform magnetic field in the x–z-plane. In the special situation of $\theta = \pi/2$, we have found that the initial state of the system plays an important role in the time evolution of the entanglement. The nonuniformity of the magnetic field has no effect on the entanglement for the $\rho_1(0) = |00\rangle\langle 00|$ case, while the uniform component of the field has no effect on the entanglement if $\rho_2(0) = |01\rangle\langle 01|$. Moreover, it is shown that the larger the anisotropic parameter ν is, the faster the concurrence will decay if $\gamma \neq 0$. On the other hand, when the magnetic field is along an arbitrary direction (except for $\theta = \pi/2$) in the x–z-plane, we have found that both the uniform component and the nonuniform component of the field have phase decoherence effects on the entanglement of the system. It is demonstrated that the introduction of anisotropy in the Z-direction will have effects on the entanglement if $\theta \neq \pi/2$ and the effects depend on the specific direction of the field. Besides, it should be pointed out that after a fixed set of the parameters ν, Δ, θ, γ, B and b is chosen, the initial larger concurrence of the two qubits will lead to higher concurrence throughout the whole evolution of the entanglement in the presence of phase decoherence.

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